There's a Lot of Powerful and Accessible Research that Needs to Find Its Way into All Mathematics Classrooms

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Steve Leinwand
American Institutes for Research
sleinwand@air.org www.steveleinwand.com

Today's Goal

To engage you in thinking about (and then being willing and able to act on) the critical and long-standing issue of supporting the transfer of what we know into widespread classroom practice.

So let’s begin with what we know:

- What does it take?
- What do we face?
- What do we know?

In a nutshell, we know what it takes

What we strive for: **LEARNING** **SUCCESS**

The enabling conditions: **ENGAGEMENT**
PARTICIPATION

What it takes: **TASKS** **QUESTIONS** **DISCOURSE**
MISTAKES **PRODUCTIVE STRUGGLE**

Enhanced by: **REPRESENTATIONS**
ALTERNATIVES
ESTIMATES

All guided by: **A CLARITY OF GOALS**

A Progression of Insights on Where we Stand

- **We are charged with making math work for a much greater proportion of students.**
- **But typical instructional practice of showing, telling and practicing to get "right answers" only works for about 1/3 of our students.**
- **To complicate matters, today's world requires reasoning, solving problems, constructing viable arguments (SMPs).**
- **Thus math classes must reflect a different set of instructional practices – productive struggle, alternative approaches and multiple representations, discourse, explanations, conjectures and justifications (MTPs).**
- **But, this is different, difficult, requires time and risk-taking.**
- **Which is why we must have a shared vision, high quality common unit assessments, collaborative structures and coaching to support envisioning, practicing and providing feedback as we raise quality and impact.**

The key things we know

People won't do what they can't envision,

People can't do what they don't understand,

People can't do well what isn't practiced,

But practice without feedback results in little change, and

Work without collaboration is not sustaining.

Ergo: Our job, as professionals, at its core, is to help people envision, understand, practice, receive feedback and collaborate.

READY?

- What does research-affirmed practice look like?
- How do we make these elements clear and accessible?

What do you notice?

Omaha
244 miles

Car Manual
Gas Tank Capacity:
21 gallons

Average MPG:
28.3

What's the question?

Omaha
244 miles

Car Manual
Gas Tank Capacity:
21 gallons

Average MPG:
28.3

Will you make it?

Omaha
244 miles

Car Manual
Gas Tank Capacity:
21 gallons

Average MPG:
28.3

About how far will you get?

Omaha
244 miles

Car Manual
Gas Tank Capacity:
21 gallons

Average MPG:
28.3

Turn and Talk: What made that experience effective?

Elements of Quality

- **Clarity of goals (not Lesson 4.5 or pages 214-217)**
- **Context (not naked)**
- **Rich tasks (not exercises)**
- **Focused intentional questions (not punting)**
- **Opportunities for discourse (not just telling)**
- **Gradual release (not just a dumping)**
- **Multiple representations (not one way)**
- **Alternative approaches (not one way)**
- **Sense-making by students (not lecture)**
- **Evidence (not I taught it and let the chips….)**

Grade 6 SA Harlem Central
Tues Dec 8, 2015

- **Lesson 6 in the Expressions unit (6.EE standards)**
- **Ally and Mabubar co-teaching**
- **19 Scholars**
- **Driven by a number strings mini-lesson, a Math Workshop task and an exit ticket**
- **“Our goal for today is to ‘identify, create and understand equivalent expressions’.”**
- **“Zayasia, can you please repeat our learning goal?”**
- **“Let’s begin with out number strings.”**

Number strings for today's Mini Lesson

Are they equivalent? How do you know?

1. **4(8) = 4(3 + 5)**

2. **4(8) = 4(a + 5)**

3. **4(8) = 4(3 + b)**

4. **3x + 3y = 3(x + y)**

Let's summarize: For each: Always, sometimes, never equivalent?

Math Workshop Task

Jan normally rides her bike to and from work.

Her normal route is 18 miles from home to work.

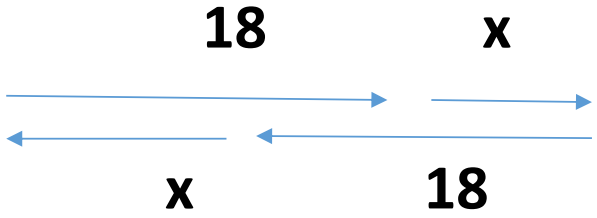
One day she goes to a coffee shop on her way to work and on her way home.

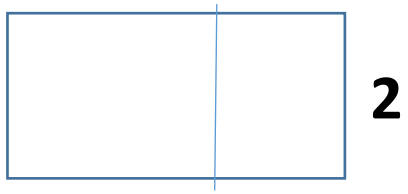
This adds x miles to her trip each way.

(“What do you notice?” “What’s the question?”)

Great: Write and show the distance Jan travels using a diagram or picture and two different, but equivalent, expressions.

Resulting in:





- **18 + x + 18 + x**
- **2(18 + x)**
- **2(x + 18)**
- **x + x + 18 + 18**
- **36 + 2x**

“Is everyone correct? Turn and tell your partner why?”

“What do the numbers and variables represent?”

“Which expression is simplest or easiest to use? Why?”

Lesson 6 Exit Ticket

Which of the following represent equivalent expressions?

Explain or show your process of determining which expressions ARE equivalent.

Select all that apply:

a. $x + x + x + x = 4x$

b. $15\ y + 5x = 5(3y + x)$

c. $6(2 + x) = 12 + 6$

d. $3(x + y) = 3x + y$

NCTM's P to A 8 MTPs

- **Establish mathematics goals to focus learning.**
- **Implement tasks that promote reasoning and problem solving.**
- **Use and connect mathematical representations.**
- **Facilitate meaningful mathematical discourse.**
- **Pose purposeful questions.**
- **Build procedural fluency from conceptual understanding.**
- **Support productive struggle in learning mathematics.**
- **Elicit and use evidence of student thinking.**

The LYNCHPIN is:

Daily classroom instruction strengthened with collaborative structures and coaching.

But what do I typically see?

- **Very little on-going cumulative review**
- **Far too many mindless worksheets**
- **Homework as the biggest waste of time**
- **Essential no exit slips or real checking for understanding and mastery**
- **Low-level questioning**
- **Little substantive student discourse**
- **Little productive struggle**
- **Inadequate use of technological solutions**
- **Far too little and far too ineffective collaboration**

Despite what the research clearing tells us.

Domain of change: On-going cumulative review

What we know:

- **Distributed practice is far more effective that clumped practice.**
- **So much of what is taught today is lost by next week or 10 minutes after the next test because it is place in short-term memory, not based on understanding and not reviewed systematically over time**
- **Look familiar:**

247	**243**	**239**
1-19 odd	**2-20 even**	**32, 34, 38**

What needs to be considered:

- **Daily on-going cumulative review**
- **2-4-2 Homework**

Number from 1 to 6

1. **What is 6 x 7?**
2. **What number is 1000 less than 18,294?**
3. **About how much is 32¢ and 29¢?**
4. **What is 1/10 of 450?**
5. **Draw a picture of 1 2/3**
6. **About how much do I weight in kg?**

Good morning Boys and Girls
Number from 1 to 5

1. What is the value of tan ($\pi/4$)?

2. Sketch the graph of $(x-3)^2 + (y+2)^2 = 16$

3. What are the equations of the asymptotes of $f(x) = (x-3)/(x-2)$?

4. If $\log_2 x = -4$, what is the value of x?

5. About how much do I weight in kg?

Cumulative Review/Distributed Practice

Almost no one masters something new after one or two lessons and one or two homework assignments. That is why one of the most effective strategies for fostering mastery and retention of critical skills is daily, cumulative review at the beginning of every lesson.

2-4-2 Homework Policies

- **2 problems on the new skill (which is usually enough to determine understanding and avoids such much practice of mistakes that it is hard to unlearn them);**
- **4 cumulative review problems roughly drawn from the day before, the week before, last month and perhaps a diagnostic readiness check for the next lesson - all of which honor distributive practice; and**
- **2 problems that require showing work or explanation and support problem solving and reasoning and justification.**

- **After beginning mathematics the next day, teachers can easily post the answers to these 8 exercises or problems on the white board and provide students with 5 minutes to review their work in pairs or triads with particular attention to the last two problems. Classroom policy can then be that correct work for any problems that are still causing trouble can be easily displayed with a document camera and discussed before homework is collected, only to be recorded as completed.**

Domain of change: Worksheets

What we know:

- biggest complaint about textbooks: NOT ENUF PRACTICE

- most independent work and lots of homework: WORKSHEETS

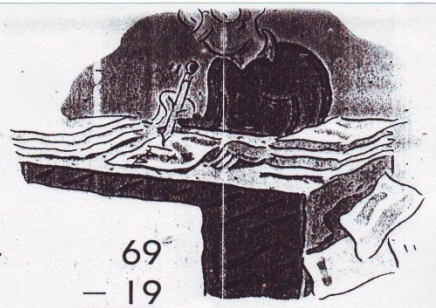
- basically answer getting with little opportunity for why, how do you know, who did it differently

What needs to be considered:

- never more than four exercises on the same skill

- the power of technology

Practice Plus



Key Skill: Subtraction, page 225

Subtract.

1.	32 − 17	48 − 23	86 − 67	54 − 31	69 − 19	
2.	77 − 46	51 − 27	98 − 25	66 − 33	40 − 16	83 − 55
3.	53 − 26	31 − 8	74 − 26	95 − 48	57 − 19	21 − 10

Key Skill: Subtracting Money, page 23

Subtract.

1.	27¢ − 18¢	31¢ − 26¢	44¢ − 27¢	63¢ − 29¢	97¢ − 59¢	80¢ − 41¢
2.	49¢ − 22¢	74¢ − 59¢	89¢ − 39¢	56¢ − 37¢	65¢ − 48¢	92¢ − 15¢
3.	38¢ − 29¢	52¢ − 13¢	84¢ − 68¢	67¢ − 43¢	75¢ − 27¢	99¢ − 11¢

Problem Set 6-2

Quick Review

5 min

Q1. Integrate: $\int x^{-0.3}\,dx$

Q2. Integrate: $\int_0^3 x^2\,dx$

Q3. Differentiate: $f(x) = \cos^2 x$

Q4. $\lim_{x\to 100} \cos x = \cos 100$, so cos is —?— at $x = 100$.

Q5. If $f(x) = x^3$, then $f'(2) = 12$. Thus f is —?— at $x = 2$.

Q6. Find y': $y = \sin^{-1} x$

Q7. Find y': $y = \csc x$

Q8. $\sum f(x)\,\Delta x$ is called a(n) —?—.

Q9. $\int f(x)\,dx$ is called a(n) —?—.

Q10. $\log 3 + \log 4 = \log$ —?—.

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0. *Look Ahead Problem 1:* Look at the derivatives and the integrals in Problem Set 6-6. Make a list, by problem number, of those you currently know how to do.

For Problems 1–26, find the derivative.

1. $y = \ln 7x$
2. $y = \ln 4x$
3. $f(x) = \ln x^5$
4. $f(x) = \ln x^3$
5. $h(x) = 6 \ln x^{-2}$
6. $g(x) = 13 \ln x^{-5}$
7. $r(t) = \ln 3t + \ln 4t + \ln 5t$
8. $v(z) = \ln 6z + \ln 7z + \ln 8z$
9. $y = (\ln 6x)(\ln 4x)$
10. $z = (\ln 2x)(\ln 9x)$
11. $y = \dfrac{\ln 11x}{\ln 3x}$
12. $y = \dfrac{\ln 9x}{\ln 6x}$
13. $p = (\sin x)(\ln x)$
14. $m = (\cos x)(\ln x)$
15. $y = \cos(\ln x)$
16. $y = \sin(\ln x)$
17. $y = \ln(\cos x)$ (Surprise?)
18. $y = \ln(\sin x)$ (Surprise?)
19. $T(x) = \tan(\ln x)$
20. $S(x) = \sec(\ln x)$
21. $y = (3x + 5)^{-1}$
22. $y = (x^3 - 2)^{-1}$
23. $y = x^4 \ln 3x$
24. $y = x^7 \ln 5x$
25. $y = \ln(1/x)$
26. $y = \ln(1/x)^4$

For Problems 27–46, integrate.

27. $\int (7/x)\,dx$
28. $\int (5/x)\,dx$
29. $\int \dfrac{1}{3x}\,dx$
30. $\int \dfrac{1}{8x}\,dx$
31. $\int \dfrac{x^2}{x^3 + 5}\,dx$
32. $\int \dfrac{x^5}{x^6 - 4}\,dx$
33. $\int \dfrac{x^5\,dx}{9 - x^6}$
34. $\int \dfrac{x^3\,dx}{10 - x^4}$
35. $\int \dfrac{\sec x \tan x\,dx}{1 + \sec x}$
36. $\int \dfrac{\sec^2 x\,dx}{1 + \tan x}$
37. $\int \dfrac{\cos x\,dx}{\sin x}$
38. $\int \dfrac{\sin x\,dx}{\cos x}$

And we need to ask:

Where is instantaneous, non-judgmental feedback?

Where, as Jo Boaler would ask, is anything that "stimulates neural activity" also known as learning?

Where are hints when errors are made?

Where do students get to manipulate and construct as they now do on the PARCC and SBAC assessments?

Where does the worksheet read the problem to students who need such an accommodation?

Where is an adaptive feature that differentiates?

And where are instantaneous scores and reports by item, by student and by class?

Domain of change: Exit slips

What we know:

- most of the answers called out or volunteered come from only about 1/3 of any given class;

- when we ask “what evidence of learning do you have?” we usually go mute or punt.

What needs to be considered:

- with 5 minutes to go, every lesson:

- “Turn and tell you partner what you learned today”;

- “Individually, on a sticky note, complete this task”;

- Launch tomorrow’s lesson with “On the basis of yesterday’s exit slip

How hard it that? Why is it so rare?

Domain of change: Questioning

What we know:

- far too many of the questions we ask are DOK Level 1 and do not induce thinking and reasoning;

- “Why” makes all the difference in the world

Domain of change: Discourse

What we know:

- teachers talk far more that students, often to the detriment of learning;

- discourse is a visible sign of engagement and engagement is a prerequisite for learning.

What needs to be considered:


WHY?
How do you know?
Convince me.
Explain that please.
Draw a picture.
Table
Graph
Johnniece King
ExponEntial
By: Ashli, Arel,
y=20+4x

Classroom Discourse

- **Why?**
- **How do you know?**
- **Can you explain that? (and who did it differently?)**
- **Convince us.**
- **How did you picture that? (and who did it differently?)**
- **What do you notice?**
- **What do you wonder?**
- **How are they the same?**
- **How are they different?**

A tale of two approaches

I

We

You

A tale of two approaches

I

We

You

So simple

So familiar

So appealing

So wrong for so many!

A tale of two approaches

I	**You**
We	**We**
You	**I**

A tale of two approaches

You	**Struggle**
	Explore
	Share
We	**Justify**
	Compare
	Debrief
I	**Consolidate**

Domain of change: Struggle

What we know:

- there is too much teaching by telling;

- there is too much spoon-feeding HOW to get answers;

- there isn’t enough productive struggle to solve problems and truly exercise the neural connections.

What needs to be considered:

Ready!

At Central Middle School, there are 3 scholars in the Science Club for every 8 scholars in the Math Club.

If there are a total of 45 scholars in one or both of these clubs, how many scholars are in both clubs?

Domain of change: Technology

What we know:

- Every other enterprise in our society has found ways to use technology to move from crawling ineffectively and inefficiently to flying in efficient and effective ways that make life better, fuller and richer. It's time we did the same for mathematics.

What needs to be considered:

- Class twitter accounts to capture and display;

- Document cameras to quickly share and display;

- Web-based resources (see attached list)

Collaborative Structures

- **Collegial classroom visits and debrief discussions**
- **Task analysis**
- **Collaborative planning**
- **Co-teaching and co-planning**
- **Common readings**
- **Lesson study**
- **Instructional rounds**
- **Analysis of student work**
- **Data reviews and actions**
- **Video analysis**
- **Learning communities**
- **Gallery teaching**
- **Common problem resolution discussions and plans**

So what begins our research-affirmed to-do list?

- **Much more on-going cumulative review**
- **Far fewer mindless worksheets**
- **Rational 2-4-2 homework**
- **Daily exit slips and real checking for understanding and mastery**
- **High-order questioning**
- **Much more substantive student discourse**
- **Much more productive struggle**
- **Far greater use of technological solutions**
- **Far more and far more effective collaboration**

ERGO:

- **We have the answers**
- **We know what it will take**
- **Things are better than they have been**
- **The challenges are greater than ever**
- **The ball is in our court**
- **The ball is your court**
- **The kids deserve no less and they are counting on us.**

Your turn:

Where will you start individually?

Where will you start collectively?

Thank you.